

$(g - 2)_\mu$ in the custodially protected RS model

P. MOCH^a and J. ROHRWILD^b

^a*Physik Department T31,
James-Franck-Straße 1, Technische Universität München,
D-85748 Garching, Germany*

^b*Rudolf Peierls Centre for Theoretical Physics,
University of Oxford, 1 Keble Road,
Oxford OX1 3NP, United Kingdom*

Abstract

We study leptonic operators of dimension six in the custodially protected Randall-Sundrum model. Their contribution to the anomalous magnetic moment of the muon is evaluated. We find that the contribution to $g-2$ due to diagrams with an internal gauge boson exchange is given by

$$\Delta a_\mu^g \approx 2.72 \times 10^{-10} \left(\frac{1 \text{ TeV}}{T} \right)^2$$

basically independent of the model parameters except for the KK mass scale T —a factor of more than 3 larger than in a scenario without custodial protection. We also investigate the impact of contributions to the dipole operators due to an internal Higgs exchange, which can provide sizable albeit model-dependent corrections.

1 Introduction

The anomalous magnetic moment of the muon is one of the most precisely predicted and measured observables in particle physics. Over the last 65 years tremendous effort has been invested in determining the deviation of the g-factor from two, see [1] for a recent review. The extraordinary precision naturally imposes constraints on extensions of the Standard Model (SM), whose contribution Δa_μ to the anomalous magnetic moment

$$a_\mu = \frac{g_\mu - 2}{2} = a_\mu^{\text{SM}} + \Delta a_\mu \quad (1)$$

should not increase the observed slight tension between SM and experimental value for a_μ .

Among the more intensively studied classes of models beyond the SM are those that extend the dimension of space-time. A particularly rich phenomenology is realised in models that live on a compact slice of a five-dimensional Anti-de-Sitter space. In conformal coordinates the invariant interval of this space-time is given by

$$ds^2 = \left(\frac{1}{kz} \right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) , \quad (2)$$

where k is of the order of the Planck mass M_{Pl} and the finite fifth dimension stretches from $z = 1/k$ to $z = 1/T$. The a priori arbitrary scale T is chosen to be of the order of a TeV to alleviate the gauge-gravity hierarchy problem [2]. While the original Randall-Sundrum (RS) set-up [3] considered an extra-dimension that is only accessible to gravity, making the five-dimensional bulk accessible to SM fields [4–7] offers a geometric interpretation of the hierarchies for masses and flavour mixing angles in the fermion sector [5, 7, 8]—only the Higgs needs to be confined near $z = 1/T$ (IR brane).

The most immediate consequences of a compact extra-dimension is the presence of higher excited modes for each SM field allowed to propagate in the bulk of AdS_5 . The mass scale of these Kaluza-Klein (KK) resonances is set roughly by $2.5 \times T$. Direct production currently only yields lower limits on T of the order of ≈ 1 TeV [11]. Indirect constraints from flavour and electroweak precision observables give far stronger bounds. To avoid, in particular, large corrections to the ρ parameter and the Zbb vertex one can extend the gauge group in the bulk to $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_X \times \mathbb{Z}_2$ with appropriate breaking mechanisms on the branes to introduce a custodial protection into the model [9, 10, 13, 14].

In the last few years, there have been increasing efforts to study the effects of the RS model not only in processes that are dominated by tree-level effects, but also via observables that are sensitive to loop-induced processes. Here, processes that are not expected to exhibit a UV sensitivity have been most promising; notable studies include e.g. Higgs production and decay [15–18], lepton [19] and quark flavour violation [20–22]. The anomalous magnetic moment in the minimal RS model has been studied recently in a 5D formalism in [23]. In this letter we will extend the results of [23] to the custodially protected RS model. Here it should be noted that the model itself is not UV complete.

The effect of the unknown UV completion on the anomalous magnetic moment is determined by the warped cut-off Λ which is of the order of several times T . Generically one expects the correction to scale as $1/\Lambda$ and thus, provided that no additional enhancement factors are present, to be subleading.

This work is organised as follows. In Section 2 we describe the custodial protected RS model and give the properties of the various bulk fields. Our strategy for the calculation of $(g - 2)_\mu$ via matching the 5D theory onto an effective 4D lagrangian is detailed in Sect. 3. We follow [23] and determine the Wilson coefficients of the relevant $SU(2) \times U(1)$ invariant dimension-six operators in a manifestly five-dimensional formalism. The various contributions are classified according to their dependence on the Yukawa structure of the 5D theory. Our numerical results for the anomalous moment are presented in Section 4. We discuss their dependence on the parameters of the 5D Lagrangian and the impact of corrections to other lepton observables. We conclude in Section 5.

2 The custodially protected model

The standard way to implement the custodial protection mechanism in the RS model is by extending gauge group in the bulk of AdS to

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times \mathbb{Z}_2$$

where the discrete \mathbb{Z}_2 enforces a symmetry under $SU(2)_L \leftrightarrow SU(2)_R$. This assures that the gauge coupling constants for both $SU(2)$ group are identical, $g_L = g_R = g$. In order to obtain the Standard Model gauge group and particle content in the low energy limit, the bulk symmetry must be broken on both branes. On the UV brane the breaking proceeds by boundary conditions, on the IR brane by via the Higgs vacuum expectation value (vev). The presence of the $SU(2)_R$ group prohibits large correction to the ρ parameter, while the discrete symmetry protects the Zbb vertex from large corrections due to the (IR localised) top, provided that the quarks are arranged in certain multiplets [10].

There is, in principle, no need to mirror this construction in the lepton sector as the corrections to the $Z\tau\tau$ vertex are generally mild. One of the more economic constructions would put the lepton doublets in a $(2, 1)$ multiplet under $SU(2)_L \times SU(2)_R$ and the singlets in a $(1, 2)$ [12]. This realisation does not provide any genuinely new diagram topologies compared to minimal RS model and no significantly enhanced contribution to $g_\mu - 2$ can be expected¹. We, therefore, turn to a model where the lepton sector is a (colourless) copy of the quark sector [25].

A detailed discussion of the model can be found in [13, 26]. For our purposes it is enough to specify the action and the boundary conditions for the various fields on the branes; we follow the conventions of [26]. The action of the custodially protected model

¹A simple estimate shows that the gauge contribution to $g_\mu - 2$ is enhanced only by a factor of about 25%.

is given by

$$\begin{aligned}
S_{(5D)} = & \int d^4x \int_{1/k}^{1/T} dz \sqrt{G} \left\{ -\frac{1}{4} X^{MN} X_{MN} - \frac{1}{4} W_R^{a,MN} W_{R,MN}^a - \frac{1}{4} W_L^{a,MN} W_{L,MN}^a \right. \\
& + \sum_{\psi=\xi_1, \xi_2, T_4, T_3} \left(e_m^M \left[\frac{i}{2} \bar{\psi}^f \Gamma^m (\vec{D}_M - \overleftarrow{D}_M) \psi^f - c_{\psi_f} k \bar{\psi}_f \psi_f \right] \right) \Big\} + S_{GF+ghost} \\
& + \int d^4x \left\{ (D^\mu \Phi)^\dagger D_\mu \Phi - V(\Phi) - \left(\frac{T^3}{k^3} \right) \mathcal{L}_{\text{Yukawa}} \right\}
\end{aligned} \tag{3}$$

where $G = 1/(kz)^{10}$ and $e_m^M = kz \text{diag}(1, 1, 1, 1, 1)$. The Higgs potential is given by

$$V(\Phi) = -\mu_{(5D)}^2 \left(\frac{T}{k} \right)^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2. \tag{4}$$

The covariant derivative is defined as

$$D_M = \partial_M - ig_X Q_X X_M - ig_5 T_L^c W_{LM}^c - ig_5 T_R^c W_{RM}^c. \tag{5}$$

Q_X is the charge under the $U(1)_X$ whereas $T_{R/L}^c$ are the generators of the $SU(2)_{L/R}$ in the appropriate representation: ξ_1 is a bi-doublet under $SU(2)_L \times SU(2)_R$, ξ_2 is a singlet and $T_{3/4}$ are singlets under $SU(2)_L$ and triplets under $SU(2)_R$. The lepton multiplets can conveniently be written in the form [13]

$$\begin{aligned}
\xi_{1L}^{il} &= \begin{pmatrix} \chi_L^{\nu_i}(-, +)_1 & l_L^{\nu_i}(+, +)_0 \\ \chi_L^{l_i}(-, +)_0 & l_L^{l_i}(+, +)_{-1} \end{pmatrix} \\
\xi_{2R}^{il} &= \nu_R(+, +)_0 \\
\xi_{3R}^{il} &= T_{3R}^i \otimes T_{4R}^i = \begin{pmatrix} \tilde{\lambda}_R^i(-, +)_1 \\ \tilde{N}_R^i(-, +)_0 \\ \tilde{L}_R^i(-, +)_{-1} \end{pmatrix} \otimes \begin{pmatrix} \lambda_R^i(-, +)_1 \\ N_R^i(-, +)_0 \\ E_R^i(+, +)_{-1} \end{pmatrix},
\end{aligned} \tag{6}$$

where the subscript of the different fields indicates the electrical charge $Q = T_L^3 + T_R^3 + Q_X$. The signs in parentheses are the boundary conditions on the UV (left) and IR brane (right); a '+' corresponds to Neumann and a '-' to Dirichlet boundary conditions. Only $(+, +)$ fields have massless zero-modes. The Standard model doublet field is contained in ξ_1 , while the singlet field is included in the T^4 triplet. Note that this model additionally includes a right-handed neutrino. The field strength tensors are given by the usual expressions

$$X_{NM} = \partial_N X_M - \partial_M X_N \quad W_{R/L, NM}^a = \partial_N W_{R/L, M}^a - \partial_M W_{R/L, N}^a + g_5 \varepsilon^{abc} W_{R/L, N}^b W_{R/L, M}^c. \tag{7}$$

The SM B boson arises as the combination

$$\begin{aligned} W_R^3 &= \cos \phi Z_X + \sin \phi B \\ X &= -\sin \phi Z_X + \cos \phi B \end{aligned} \quad (8)$$

in analogy to the W_L^3 - B mixing in the SM. In particular it follows that

$$\cos \phi = \frac{g_5}{\sqrt{g_5^2 + g_{5X}^2}} \quad (9)$$

and

$$g'_5 = \frac{g_5 g_{5X}}{\sqrt{g_5^2 + g_{5X}^2}}. \quad (10)$$

for the U(1) hypercharge coupling g'_5 . The corresponding boundary conditions for the four-vector components of the gauge fields are

$$W_L^{\mu,a} (+, +), W_R^{\mu,b} (-, +) \quad (11)$$

$$B^\mu (+, +), Z_X^\mu (-, +) \quad (12)$$

with $a = 1, 2, 3$ and $b = 1, 2$.

The Yukawa interaction is described by the Lagrangian density [13]

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= \sum_{i,j=1}^3 \left\{ -(\lambda_{5d}^u)_{ij} (\bar{\xi}_1^i)_{a\alpha} \Phi_{a\alpha} \xi_2^j \right. \\ &\quad \left. + \sqrt{2} \lambda_{5d}^d_{ij} \left[(\bar{\xi}_1^i)_{a\alpha} \frac{(\tau^c)_{ab}}{2} (\tilde{T}_3^j)_c \Phi_{b\alpha} + (\bar{\xi}_1^i)_{a\alpha} \frac{(\tau^\gamma)_{\alpha\beta}}{2} (\tilde{T}_4^j)_\gamma \Phi_{a\beta} \right] + \text{h.c.} \right\} \end{aligned} \quad (13)$$

where we show all group indices explicitly. The $\tilde{T}_{3,4}$ are defined as

$$(\tilde{T}_3^i) = \begin{pmatrix} \frac{1}{\sqrt{2}} (\tilde{\lambda}^i + \tilde{L}^i) \\ \frac{i}{\sqrt{2}} (\tilde{\lambda}^i - \tilde{L}^i) \\ \tilde{N}^i \end{pmatrix} \quad (\tilde{T}_4^i) = \begin{pmatrix} \frac{1}{\sqrt{2}} (\lambda^i + E^i) \\ \frac{i}{\sqrt{2}} (\lambda^i - E^i) \\ N^i \end{pmatrix} \quad (14)$$

in the basis of (6). Note that the Higgs is not charged under the $U(1)_X$ but is a bi-doublet under the $SU(2)$ groups:

$$\Phi = \begin{pmatrix} \pi^+/\sqrt{2} & -(v + h^0 - i\pi^0)/\sqrt{2} \\ (v + h^0 + i\pi^0)/\sqrt{2} & \pi^-/\sqrt{2} \end{pmatrix}, \quad (15)$$

where $h^0 = \pi^\pm = 0$ gives the vacuum expectation value of the bi-doublet.

In writing (3) we effectively assumed that the Higgs field is delta-function localised on the IR brane and eliminated the integral over the bulk coordinate z using the $\delta(g_{55}(z - 1/T))$ factor. As has been pointed out numerous in the literature [17, 18, 21] the localisation should be implemented via some limiting procedure. To completely specify the model one needs to give a prescription how to take the limit of the regulator for the δ -distribution. We will discuss this in section 3.2.

3 Matching onto an effective dimension six lagrangian

We will follow the multi-step matching strategy prescribed in [23], i.e. we integrate out the effect of the heavy KK fields and match onto a manifestly $SU(3)_c \times SU(2)_L \times U(1)$ symmetric Lagrangian. Hence, we will be brief and refer the reader to [23] for a detailed discussion. The dominant effects beyond the Standard Model can then be captured by operators of dimension six² [27, 28].

$$\mathcal{L}^{RS_{cs}} \rightarrow \mathcal{L}^{SM} + \frac{1}{T^2} \sum_i c_i \mathcal{O}_i \quad (16)$$

where

$$\begin{aligned} \sum_i c_i \mathcal{O}_i = & a_B^{ij} \bar{L}_i \sigma^{\mu\nu} E_j \Phi B_{\mu\nu} + a_W^{ij} \bar{L}_i \tau^A \sigma^{\mu\nu} E_j \Phi W_{\mu\nu}^A + \text{h.c.} \\ & + b^{ij} \bar{L}_i \gamma^\mu L_i \bar{E}_j \gamma_\mu E_j \\ & + c_1^i \Phi^\dagger i D_\mu \Phi \bar{E}_i \gamma^\mu E_i + c_2^i \Phi^\dagger i D_\mu \Phi \bar{L}_i \gamma^\mu L_i + c_3^i \Phi^\dagger \overleftrightarrow{\tau^A D_\mu} \Phi \bar{L}_i \tau^A \gamma^\mu L_i \\ & + h^{ij} \Phi^\dagger \Phi \bar{L}_i \Phi E_j + \text{h.c.} \end{aligned} \quad (17)$$

where $\overleftrightarrow{\tau^A D_\mu} = 1/2(\tau^A \overrightarrow{D_\mu} - \overleftarrow{D_\mu} \tau^A)$. L_i represents a lepton doublet field of flavour i and E_i stands for the SM lepton singlet. The Higgs doublet is given by Φ and $B_{\mu\nu}$ and $W_{\mu\nu}^A$ are the usual field strength tensors of $U(1)_Y$ and $SU(2)_L$ gauge field, respectively. The covariant derivative is given by

$$D_\mu = \partial_\mu - ig' \frac{Y}{2} B_\mu - ig_5 T^a W_\mu^a \quad (18)$$

with Y being the hypercharge operator and T^a being the generators of $SU(2)$ in the appropriate representation. (17) contains only those operators that can either contribute to the anomalous magnetic moment at the one loop-level in the effective theory and can be generated at tree level in the full theory (last three lines), or they contribute at tree-level but are generated by loops in the 5D theory (first line).

²The single dimension-five operator is not relevant for our analysis.

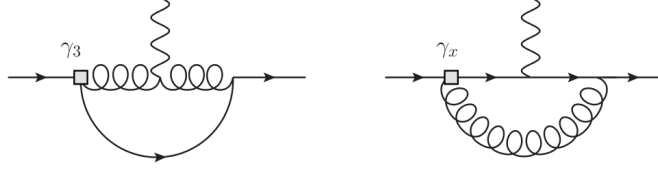


Figure 1: SM electroweak diagrams (in unitary gauge) with insertions of operators of the form $\bar{\psi}\gamma^\mu\psi \cdot \Phi^\dagger D_\mu\Phi$. Diagrams with an operator insertion in the other vertex are not shown.

The basic formula [23], equation (2.26) therein, for the anomalous magnetic moment is

$$\Delta a_\mu = -\frac{4m_\mu^2}{T^2} \left(\frac{\text{Re}(\alpha_{22})}{y_\mu e} + \sum_{k=1,2,3} \frac{1}{16\pi^2} \frac{m_{\ell_k}}{m_\mu} \text{Re}(\beta_{2kk2}) \right) \quad (19)$$

with

$$\begin{aligned} \alpha_{ij} &= [U^\dagger a V]_{ij}, \\ a_{ij} &= \cos \Theta_W a^{B,ij} - \sin \Theta_W a^{W,ij}, \\ \beta_{ijkl} &= \sum_{m,n} [U^\dagger]_{im} U_{mj} [V^\dagger]_{kn} V_{nl} b^{mn}, \end{aligned} \quad (20)$$

where U, V are the rotation matrices into the mass eigenbasis for charged doublet and singlet leptons, respectively. The contribution proportional to the Wilson coefficient h is not shown as it is suppressed by a factor of m_ℓ/v . (19) does not include effects that can directly or indirectly be associated with modifications of SM parameters. In particular, the direct modification of the Z-muon and W-muon couplings³ via the Wilson coefficients $c_{1,2,3}^i$ was omitted. They enter via the diagrams shown in Fig. 1 which arise from insertions of dimensions-six operators in SM one-loop diagrams. Their effect on $g-2$ is given by

$$\Delta a_\mu^{ZW} = -\frac{4m_\mu^2}{T^2} \frac{1}{3(4\pi)^2} \text{Re} \left[\gamma_2^{22} - \frac{3}{2} \gamma_1^{22} - \frac{3}{2} \gamma_3^{22} + \sin^2 \Theta_W \sum_{i=1}^3 \gamma_i^{22} \right] \quad (21)$$

with

$$\gamma_1^{ij} = \sum_k [V^\dagger]_{ik} c_1^k V_{kj} \quad \gamma_x^{ij} = \sum_k [U^\dagger]_{ik} c_x^k U_{kj}, \quad x = 2, 3, \quad (22)$$

³Assuming one does not use e.g. the measured values for the W/Z-muon couplings as input parameters.

see also [32] where different conventions for operator normalisation, covariant derivative and momentum flow are used. Furthermore, there are indirect effects that stem from modifications of SM relations. For example, the SM electroweak corrections to a_μ are usually written in terms of G_F . However, the usual relation of W mass, electroweak coupling g and G_F receives corrections from dimension-six operators, see e.g. [24]. These indirect effects depend on the choice of input parameters and, while readily calculable, are omitted here.

We now can relate the Wilson coefficients in (17) to the shift in $(g-2)_\mu$. This leaves us with the determination of the Wilson coefficients in the (custodial) RS model.

3.1 Gauge Contributions

The terms contributing to the Wilson coefficients in the dimension-six Lagrangian that are at most linear in the Yukawa couplings necessarily arise from diagrams that contain at least one exchange of a 5D gauge boson. We therefore usually refer to these terms as *gauge contributions*. On the other hand there are terms that involve three Yukawa factors; these are usually generated by an exchange of a Higgs boson and hence dubbed *Higgs contributions*. We will discuss the two separately. The gauge contributions in the minimal model were discussed at length in [23]. Most of these results carry over to the custodially protected model. We only need to account for effects that originate from the additional particles in the spectrum.

In the case of operators that can be generated at tree-level in the 5D theory there are only three additional diagrams that influence the matching calculation. The reason for this is that an interaction with the new non-abelian gauge bosons $W_R^{1,2}$ always changes the $SU(2)_R$ quantum number of leptons in such a way that at least one of the fermions must not have a zero-mode. Only the Z_X boson can modify the Wilson coefficients of tree-level operators relative to their value in the minimal model. The diagrams that need to be evaluated are shown in figure 2. Note, that the fifth component of the Z_X cannot appear as the external modes at each vertex have the same handedness.

Since the external momenta are always much smaller than the KK scale T we only need the expression for the Z_X^μ propagator in the limit of vanishing 4D momentum q :

$$\Delta_\perp^{Z_X}(q \rightarrow 0, x, y) = \frac{i}{2k} [k^2 \min\{x^2, y^2\} - 1] . \quad (23)$$

This expression can be obtained from [23] by taking into account the modified boundary condition on the UV brane. In the following we give the complete expressions for the Wilson coefficients b_{ij} and c_x , $x = 1, 2, 3$. We dropped terms that are suppressed by powers of the tiny ratio $\epsilon = T/k$. The first line always gives the contribution due to B and W_L boson exchanges that are already present in the minimal model; the second line, if present, represents the contribution from the new bosons.

$$c_{1,i} = \frac{g'^2 Y_E}{8} \left(1 - \frac{1}{\ln 1/\epsilon} - \left[\frac{(1 + 2c_{E_i})(5 + 2c_{E_i})}{(3 + 2c_{E_i})^2} - \frac{2(1 + 2c_{E_i}) \ln 1/\epsilon}{(3 + 2c_{E_i})} \right] \frac{\epsilon^{-2c_{E_i}-1}}{1 - \epsilon^{-2c_{E_i}-1}} \right)$$

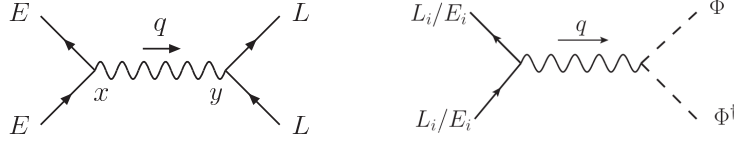


Figure 2: Additional diagrams for the tree-level matching calculation. The internal gauge boson is a Z_X .

$$+ \frac{(g^2 - g'^2)Y_E}{4} \left[\frac{(1 + 2c_{E_i}) \ln 1/\epsilon}{(3 + 2c_{E_i})} \right] \frac{\epsilon^{-2c_{E_i}-1}}{1 - \epsilon^{-2c_{E_i}-1}} \quad (24)$$

$$c_{2,i} = \frac{g'^2 Y_L}{8} \left(1 - \frac{1}{\ln 1/\epsilon} - \left[\frac{(1 - 2c_{L_i})(5 - 2c_{L_i})}{(3 - 2c_{L_i})^2} - \frac{2(1 - 2c_{L_i}) \ln 1/\epsilon}{(3 - 2c_{L_i})} \right] \frac{\epsilon^{2c_{L_i}-1}}{1 - \epsilon^{2c_{L_i}-1}} \right) \\ + \frac{(g^2 - g'^2)Y_L}{4} \left[\frac{(1 - 2c_{L_i}) \ln 1/\epsilon}{3 - 2c_{L_i}} \right] \frac{\epsilon^{2c_{L_i}-1}}{1 - \epsilon^{2c_{L_i}-1}} \quad (25)$$

and

$$c_{3,i} = \frac{g^2}{8} \left(1 - \frac{1}{\ln 1/\epsilon} - \left[\frac{(1 - 2c_{L_i})(5 - 2c_{L_i})}{(3 - 2c_{L_i})^2} - \frac{2(1 - 2c_{L_i}) \ln 1/\epsilon}{3 - 2c_{L_i}} \right] \frac{\epsilon^{2c_{L_i}-1}}{1 - \epsilon^{2c_{L_i}-1}} \right). \quad (26)$$

Y_E and Y_L are the hypercharges of singlet and doublet, respectively. The coefficient for the four-lepton operator $\bar{L}_i \gamma_\mu L_i \cdot \bar{E}_j \gamma^\mu E_j$ is given by

$$b_{ij} = b_0 + b_1(c_{L_i}) + b_1(-c_{E_j}) + b_2(c_{L_i}, c_{E_j}) \\ + \frac{g^2 - g'^2}{g'^2} b_2(c_{L_i}, c_{E_j}) \quad (27)$$

with

$$b_0 = -\frac{g'^2}{8} \frac{1}{\ln(1/\epsilon)}, \\ b_1(c) = -\frac{g'^2}{8} \frac{(5 - 2c)(1 - 2c)}{(3 - 2c)^2} \frac{\epsilon^{2c-1}}{1 - \epsilon^{2c-1}}, \\ b_2(c_L, c_E) = -\frac{g'^2}{4} \frac{(1 - 2c_L)(1 + 2c_E)(3 - c_L + c_E)}{(3 - 2c_L)(3 + 2c_E)(2 - c_L + c_E)} \ln \frac{1}{\epsilon} \frac{\epsilon^{2c_L-1}}{1 - \epsilon^{2c_L-1}} \frac{\epsilon^{-2c_E-1}}{1 - \epsilon^{-2c_E-1}}. \quad (28)$$

Note that for UV localised lepton zero-modes the Wilson coefficients c_i are dominated by the '1' in the first line. All flavour dependent terms are then subleading.

The Wilson coefficients for the dipole operators are more complicated to obtain than the Wilson coefficients generated by tree-level exchanges. We need to compute 5D one-loop diagrams. The diagram topologies are shown in figure 3, while the allowed assignments for the particles⁴ in each topology are given in tables 1 and 2. Our strategy for evaluating the 5D loops is identical to the one given in [23], section 3.3 therein, and we refer the reader to the reference for the technical details. In particular, the analytic scheme-independence and gauge-invariance proofs proceed in full analogy to the calculation in the minimal model. Still some comment on the 5D gauge parameter ξ is in order. It had to be introduced to disentangle the fifth component of the gauge field from the vector components under free propagation [29]. The Wilson coefficients of the dipole operators have to be independent of the choice for the gauge fixing term and therefore of ξ . Since we can choose different gauge parameters for, e.g., abelian and non-abelian fields we can use gauge-invariance as a separate check for the contribution of individual gauge fields. In all calculations we work in general R_ξ gauge and keep the gauge parameter ξ as a free variable. For a more detailed discussion of gauge-invariance and the importance of one-particle reducible diagrams see [30].

The main difference to the calculation in the minimal model is that the topology of a diagram is not enough to fix the structure of the integrals as we now have to deal with fermions and bosons with different possible boundary conditions on the branes. This substantially increases the computational effort of evaluating the Wilson coefficients, but does not lead to any additional conceptual difficulties compared to [23].

In [23] the loop integrals, that is the integrals over the bulk positions of each vertex as well as the integral over the modulus of the 4D loop momentum l were carried out numerically. Given the large number of diagrams this strategy is no longer feasible in the custodially protected model. To improve our accuracy we use analytic solutions for all vertex integrals with external gauge bosons based on orthogonality and completeness relations. We then only have to perform at most two bulk coordinate integrals and the l -integration numerically.

A further decrease in the numerical uncertainty can be achieved by evaluating the remaining integrals in a two step procedure. We first utilise routines provided by the CUBA library [31] to perform the remaining coordinate integrals for fixed values of the modulus of the 4D loop-momentum. We choose a set of 2180 loop-momentum values. These are used to construct an interpolation grid for the l -integrand. The first 2000 points capture the structure of this integrand in the interval $l \in (0, 10T)$. In this region we cannot use simplifications and need to exclusively rely on the numerical results. For $l > 10T$ we can already make use of the result for the integrand that is obtained if all propagators are replaced by their asymptotic forms for $l \gg T$, i.e., products of hyperbolic sine and cosine functions. In this case the coordinate integrals can be solved analytically and the l integrand has a simple form. We then use the remaining 180 points to map the deviations from the asymptotic form in the momentum region $10T < l < 100T$. For $l > 100T$ we can rely solely on the asymptotic expression. The grid together with the

⁴Note that ξ_2 cannot propagate. Hence, only the Yukawa λ^d can appear in the Wilson coefficient.

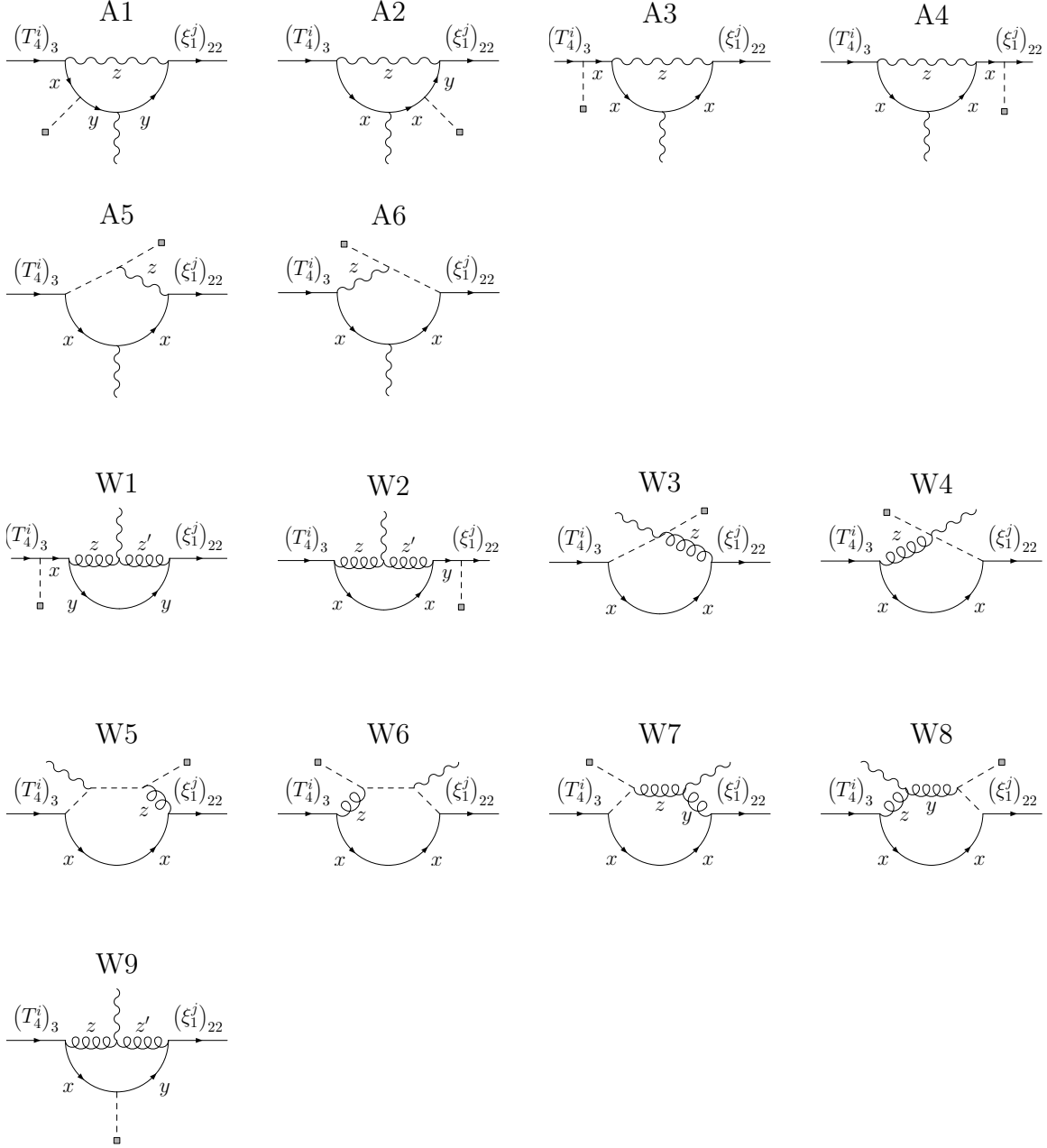


Figure 3: All topologies with internal gauge bosons that contribute to the matching of the dipole operator Wilson coefficient at one loop. Abelian topologies are labelled by A1-6, non-abelian topologies by W1-10. Fermions represented by straight lines and Higgs bosons by dashed lines. The final and initial fermions are always assumed to be $(T_4)_3$ and $(\xi_1)_{22}$ —the fields corresponding to the SM singlet and doublet. x, y, z, z' label the species of the internal propagators, see tables 1 and 2 for the allowed particles.

	x	y	z		x	y	z
A1	$(T_4^i)_3$	$(\xi_1^j)_{22}$	B^N	A2	$(T_4^i)_3$	$(\xi_1^j)_{22}$	B^N
	$(T_4^i)_3$	$(\xi_1^j)_{22}$	Z_X^N		$(T_4^i)_3$	$(\xi_1^j)_{22}$	Z_X^N
A3	$(\xi_1^j)_{22}$	/	B^N	A4	$(T_4^i)_3$	/	B^N
	$(\xi_1^j)_{22}$	/	Z_X^N		$(T_4^i)_3$	/	Z_X^N
	$(\xi_1^j)_{22}$	/	W_L^{3N}				
A5	$(\xi_1^j)_{22}$	/	B^μ	A6	$(T_4^i)_3$	/	B^μ
	$(\xi_1^j)_{22}$	/	Z_X^μ		$(T_4^i)_3$	/	Z_X^μ
	$(\xi_1^j)_{22}$	/	$W_L^{3\mu}$				

Table 1: Possible field configuration inside the loop of the abelian diagram topologies A1-A6 . A capital roman index on a gauge field indicates that both the vector and the scalar fifth component are valid options, a small Greek index shows that only the vector components may propagate.

asymptotic result are then used to construct an interpolating function of the integrand for all values of l . As a final step, this interpolating function is integrated with Mathematica.

This detour via an interpolating grid is superior to a simultaneous numerical integration of all integration variables, i.e., the bulk coordinates and modulus of the loop momentum. In general the difference between the two approaches is negligible. However, in diagrams with an exchange of a gauge boson that has a zero-mode we observe subtle cancellations on the integrand level that are only accurately resolved (for acceptable computer run-times) in the grid approach. This issue only becomes numerically relevant for 5D fermion mass parameters that force the corresponding fermion zero-modes to be localised towards the IR brane. Hence, the error introduced by the naive integration approach is generally negligible for the study of lepton observables. However, the same integral structures appear, e.g., in $b \rightarrow s\gamma$ and $t \rightarrow c\gamma$ where the effect can be of the order of several percent.

In total the optimised integration routine is by more than a factor of 10 faster and more accurate than the 'brute force' approach we used in [23].

Obviously, we cannot give the Wilson coefficient of the dipole operators in a closed analytical expression. However, it is possible to give a graphical representation of its dependence on the 5D parameters. To this end, we rewrite a_{ij}^g , the gauge contribution to a_{ij} , in the following way

$$a_{ij}^g = \lambda_{ij}^d \frac{T^3}{k^3} f_{L_i}^{(0)}(1/T) g_{E_j}^{(0)}(1/T) \mathcal{A}_{ij}, \quad (29)$$

where no summation over repeated indices is performed. The prefactors in (29) have been chosen such that \mathcal{A}_{ij} is given by a_{ij}^g with each matrix element rescaled by the corresponding entry in the lepton mass matrix. \mathcal{A}_{ij} is a function of only T , the 5D

	x	y	z	z'		x	y	z	z'
W1	$(\xi_1^j)_{22}$	$(\xi_1^j)_{12}$	W_L^{-N}	$W_L^{+N'}$	W2	$(T_4^i)_2$	$(T_4^i)_3$	W_R^{-N}	$W_R^{+N'}$
	$(\xi_1^j)_{22}$	$(\xi_1^j)_{21}$	W_R^{-N}	$W_R^{+N'}$					
W3	$(\xi_1^j)_{12}$	/	$W_L^{+\mu}$	/	W4	$(T_4^i)_2$	/	$W_R^{+\mu}$	/
W5	$(\xi_1^j)_{12}$	/	$W_L^{+\mu}$	/	W6	$(T_4^i)_2$	/	$W_R^{+\mu}$	/
W7	$(\xi_1^j)_{12}$	W_L^{+N}	$W_L^{-\mu}$	/	W8	$(T_4^i)_2$	$W_R^{+\mu}$	W_R^{-N}	/
W9	$(T_4^i)_2$	$(\xi_1^j)_{21}$	W_R^{-N}	$W_R^{+N'}$					

Table 2: Possible field configuration inside the loop of the non-abelian diagram topologies W1-W8. A capital roman index on a gauge field indicates that both the vector and the scalar component are valid options, a small Greek index shows that only the vector components may propagate.

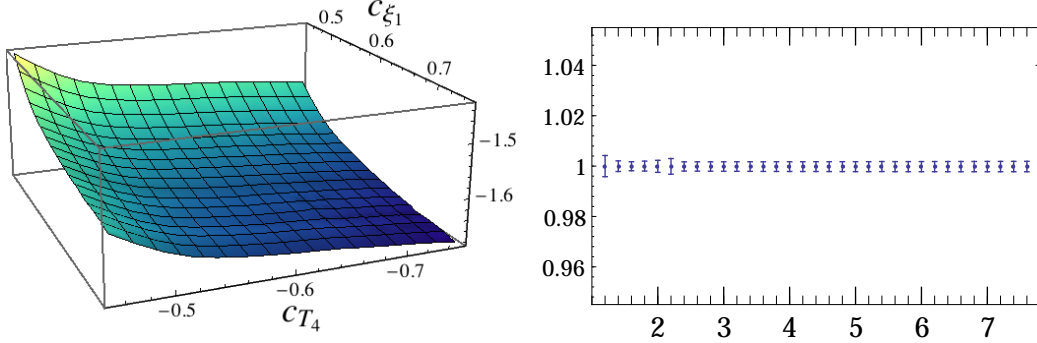


Figure 4: *left panel*: $10^8 \cdot \mathcal{A}_{ij}$ as a function of the 5D mass parameters $c_{\xi_{1i}}$ and $c_{T_{4j}}$ for $T = 1000 \text{ GeV}$, $k = 2.44 \cdot 10^{18} \text{ GeV}$. *right panel*: Residual dependence of a_{ij} on the gauge parameter ξ normalised on the value of a_{ij} for $\xi = 2$ (the error bars indicate the numerical uncertainties as estimated by our integration routines [J]).

mass parameters $c_{T_{4i}}$ and $c_{\xi_{1j}}$ and the Planck scale k . In particular, we can interpret \mathcal{A}_{ij} as a measure for the misalignment of the Wilson coefficient a_{ij}^g relative to the mass matrix *before* rotation into the mass eigenbasis. If, for fixed T and k , \mathcal{A}_{ij} is a constant in mass parameter space there is no misalignment to leading order in v^2/T^2 and the dipole operator is not lepton-flavour violating. Conversely, a strong dependence on the c-parameters indicates sizable FCNCs after EWSB. Figure 4 (left panel) shows our result for \mathcal{A}_{ij} . The dependence on the 5D bulk masses is mild ($\pm 10\%$ for the typical range of mass parameters). Hence, we can expect the anomalous magnetic moment to be basically independent of the 5D Yukawa structure and the dipole operator to induce only small flavour-violating transitions. It should be noted that the relative variation of \mathcal{A}_{ij} is by roughly a factor of two larger than in the minimal model, while its magnitude changed by a factor of three.

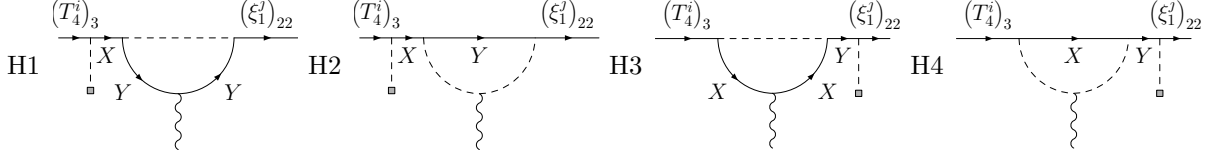


Figure 5: All topologies with internal Higgs bosons that contribute to the matching of the dipole operator Wilson coefficient at one loop. X and Y label the species of the internal propagators. The only assignments that do not cause the diagrams to trivially vanish are: $X = \xi_1$ for all diagrams and $Y \in \{T_3, T_4\}$ for H1, H2 and H4 as well as $Y \in \{\xi_2, T_3, T_4\}$ for H3.

To check our result and to obtain an independent estimate on our numerical precision one can study the residual dependence on the 5D gauge parameter ξ . The right panel in figure 4 shows a_{ij}^g as a function of ξ normalised on the value for $\xi = 2$ for the parameter set $(T, c_L, c_E) = (1 \text{ TeV}, 0.5478, -0.5478)$. The relative variation of our results with the gauge parameter is below 1 per mille. This is smaller than the typical error estimate ($\sim 2\%$) provided by the numerical integration routine. In Feynman gauge ($\xi = 1$) the numerical uncertainties are even smaller (below 1%), as the integrand takes a particularly simple form.

3.2 Higgs Contributions

The previously discussed gauge contributions are, at least to leading order in the couplings, not sensitive as to how the Higgs localisation is regularised [23]. This is not the case for contributions that are not linear in the Yukawa couplings. In the following, we use the simple regularisation

$$\delta(z - 1/T) = \lim_{\delta \rightarrow 0} \frac{T}{\delta} \Theta(z - \frac{1 - \delta}{T}). \quad (30)$$

To fix our model we also need to specify in which order the regulator δ and the regulator for the loop integrals, e.g., the dimensional regulator ϵ or a cut-off are taken to their physical values. It should be noted that the Higgs profile (30) introduces the new scale T/δ into the theory. All diagrams that are sensitive to the precise way in which the Higgs is localised have to be analysed with this new momentum region in mind, see [23, 30] for detailed examples.

The diagrams contributing to the dipole operators are shown in figure 5. Their short-distance part can be evaluated analytically for the choice (30). We use dimensional regularisation for the $(4 - 2\epsilon)$ dimensional momentum integrals; the integrals over the fifth dimension are then trivial.

If we choose to keep the δ regulator finite until all other regulators have been removed from the theory, we obtain

$$a_{ij}^H = \frac{T^{11}}{2k^{12}} \frac{Q_\mu e}{192\pi^2} \left[f_{L_i}^{(0)}(1/T) Y_{ik}^d g_{E_k}^{(0)}(1/T)^2 Y_{kh}^{\dagger, d} F(c_{\xi_{1h}}) Y_{hj}^d g_{E_j}^{(0)}(1/T) \right]$$

$$\begin{aligned}
& + 2f_{L_i}^{(0)}(1/T)Y_{ik}^u F(c_{\xi_{2k}})Y_{kh}^{u,\dagger} f_{L_h}^{(0)}(1/T)^2 Y_{hj}^d g_{E_j}^{(0)}(1/T) \Big] \\
& + \frac{T^3}{k^4} \frac{Q_\mu e}{192\pi^2} \left[f_{L_i}^{(0)}(1/T)[Y^d Y^{\dagger,d} Y^d]_{ij} g_{E_j}^{(0)}(1/T) - f_{L_i}^{(0)}(1/T)[Y^u Y^{u,\dagger} Y^d]_{ij} g_{E_j}^{(0)}(1/T) \right]
\end{aligned} \tag{31}$$

with

$$F(c) = \frac{k^4((1+2c) + (3-2c)\epsilon^{2-4c} - \epsilon^{1-2c}(3+4c-4c^2))}{(4c^2-4c-3)T^5(1-\epsilon^{1-2c})^2} \tag{32}$$

for the Higgs contributions to the dipole operator. $Y^{d/u} = \lambda_{(5D)}^{d/u} k$ are dimensionless 5D Yukawa matrices and we dropped terms that are suppressed by powers of the small ratio ϵ . Note the setting the Yukawa Y^u that arises via the $\bar{\xi}_1 \Phi \xi_2$ term in the Lagrangian to zero reproduces the result from the minimal model. This is due to an accidental cancellation of the various new diagrams.

If we choose to send the width of the Higgs to zero *before* the regulator of the 4D loop integral, the last line in (31) vanishes identically, see also [23]. In this situation the dipole operator induced by Higgs exchanges is typically much smaller than in the case with a reversed order of the limits as the third line in (31) is numerically dominant in almost all points of the parameter space.

As in case of the gauge contributions, the result for the dipole coefficient is scheme dependent. It depends on the treatment of γ_5 in $4-2\epsilon$ dimensions; (31) corresponds to naive dimensional regularisation (NDR). The scheme dependence is only cancelled when including the contributions of the operators $\Phi^\dagger \Phi \bar{L}_i \Phi E_j$ and $\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi \bar{E}_i \gamma^\mu E_j$ whose contributions vanish to our order in the expansion in NDR, but are finite in other schemes.

4 Numerical Result

With the Wilson coefficients at hand, we can now discuss the modification of the anomalous magnetic moment in the custodially protected RS model. The main input parameters are the 5D Yukawa matrices and the 5D masses. As the theory has to reproduce the SM lepton sector in the low energy limit we are provided with the additional constraints via lepton masses and mixing. For simplicity, we only require that the charged lepton masses are reproduced, all (Dirac) neutrino masses are below 0.1 eV and that their mass splitting does not violate the bounds from neutrino oscillation; we do not require that the PMNS matrix is reproduced. It should be noted that the dependence on $c_{\xi_2}^i$ (which only enters in terms with at least three Yukawa factors) is quite small. We randomly generate parameter sets that pass all constraints and determine a_μ for each set. Following the general spirit of the RS model our Yukawas are anarchic, i.e. each matrix element is very roughly of modulus one and has a random phase. In practice, we allow for values from 1/10 to 10 for the absolute value and use a flat distribution for its logarithm.

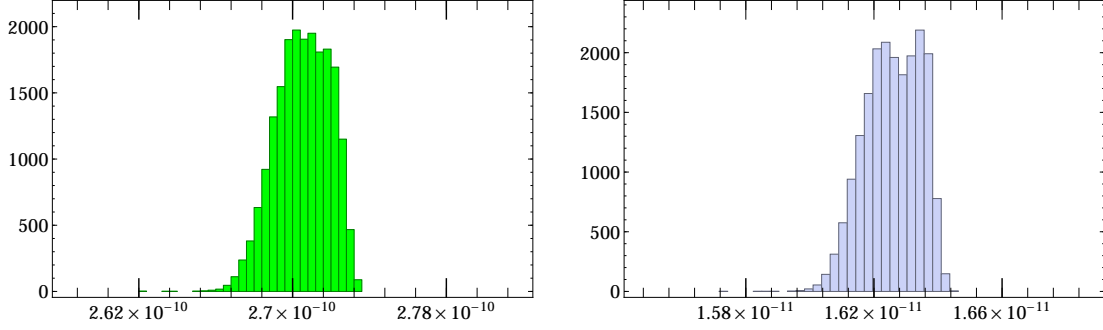


Figure 6: *left panel:* Histogram of the contribution of the gauge diagrams Δa_μ^g to the anomalous magnetic moment a_μ for fixed $T = 1$ TeV. *right panel:* Same for $T = 4$ TeV.

The gauge contributions are expected to be virtually independent of our parameter choice, as \mathcal{A}_{ij} , cf. (29), is approximately mass-parameter independent. The left panel of figure 6 shows the result for Δa_μ^g for a fixed value of $T = 1$ TeV. The result for $T = 4$ TeV is shown in the right panel. The histograms are generated from each 10^5 parameter sets. The distribution is centred around $2.72 \cdot 10^{-10}$ for $T = 1$ TeV while $T = 4$ TeV lead to a central value of $1.63 \cdot 10^{-11}$. This is in line with the typical scaling $\Delta a_\mu^{\text{gauge}} \propto \frac{1}{T^2} \times \ln k/T$. The model-independent gauge contribution to $g_\mu - 2$ in the custodially protected RS model can thus be reliably estimated via

$$\Delta a_\mu^g \approx 2.72 \times 10^{-10} \left(\frac{1 \text{ TeV}}{T} \right)^2 \quad (33)$$

for any phenomenologically relevant value of T .

Comparing to the result in the minimal model [23], $\Delta a_\mu^{\text{min}} \approx 0.88 \cdot 10^{-10} (1 \text{ TeV})^2 / T^2$ we see the minimal model gives a correction to the anomalous magnetic moment that is roughly a factor of 3 smaller, while the T dependence is, as expected, the same. Despite the significant enhancement compared to the minimal model, more realistic choices of $T > 2000$ GeV (which corresponds to KK masses larger than 4.7 TeV) only gives an enhancement to a_μ of at best $6.8 \cdot 10^{-11}$. The difference between the current experimental value and the SM prediction for the anomalous magnetic moment of the muon is given by [1]

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 287(63)(49) \times 10^{-11} \quad (34)$$

where theory and experimental uncertainties are given separately. Thus, the gauge contribution Δa_μ^g to a_μ alone is too small to be noticed in experiments.

The effect of the modified W/Z coupling Δa_μ^{ZW} is not included in the above numbers. For mass parameters $|c_{L/E}| > 0.55$ it is given by

$$\Delta a_\mu^{ZW} \approx -0.46 \cdot 10^{-11} \left(\frac{1 \text{ TeV}}{T} \right)^2 \quad (35)$$

and is, for general 5D masses of the order of $\text{few} \times 10^{-12} \frac{1 \text{ TeV}^2}{T^2}$ in both the minimal and custodially protected model. This is negligible for the custodially protected and a $\sim 5\%$ correction in the minimal model.

The Higgs contributions are strongly dependent on the model parameters, especially the Yukawa matrices. So general statements as in the case of the gauge contribution are not feasible. However, it is worthwhile to study the effect of the Higgs exchange in several illustrative scenarios. As only one of the two Higgs localisations discussed in section 3.2 gives rise to sizable corrections to the dipole operators the following discussion will focus on this case only.

Let us first go back to the minimal RS model which was already discussed in [23]. In this case we only need to consider the first term in each square bracket in (31). Obviously, the contribution to $g_\mu - 2$ will increase with the magnitude of the Yukawa matrix (in the minimal case there is only one lepton Yukawa). To quantify this statement we study the shift of $(g - 2)_\mu$ due to the dipole Wilson coefficient a^H for three hypothetical cases: the Yukawa entries are each in the range $(1/10, 1/3)$, $(1/3, 3)$ or $(3, 10)$, T is fixed to 1 TeV and we generate 10^4 random data sets for each scenario. Figure 7 (left panel) shows the result for the different Yukawa ranges using a logarithmic scale for the abscissa. One can see that the central values of the histograms scale with the square of the corresponding average Yukawa size. This was to be expected from (31) as the product of zero-mode profiles compensates for one Yukawa factor provided the Yukawa matrices themselves do not carry a strong hierarchy. As each of the distributions is spread out over more than an order of magnitude it is not possible to make quantitative statements without a detailed knowledge of the Yukawa matrices. We also find that a^H favours a positive contribution to $(g - 2)_\mu$ if one constrains the Yukawas as described. Here the logarithmic scale on the x-axis is slightly misleading: it illustrates the scaling with the Yukawa size but misses a short tail in the negative region. Nonetheless, the contributions are predominantly positive. This is interesting as the Higgs contribution is then aligned with the gauge contribution: both reduce the difference between theory and experiment (34). We can use the current limits on $g_\mu - 2$ to give a rough bound on the ratio $\frac{\langle YY^\dagger \rangle}{T^2}$. The bound is, in a sense, maximally weak, as the preference for a positive sign forces us to consider $\Delta a_\mu^{RS} < 6 \cdot 10^{-9}$ as an upper bound. Thus the constraining power of $g_\mu - 2$ for the lepton Yukawa sector is weaker than Higgs production [16] is for the quark sector even though both are sensitive to the same ratio $\frac{\langle YY^\dagger \rangle}{T^2}$.

Only average Yukawa entries of at least 3 would allow for a correction that is sizable enough to remove the current tension. However, such large values would, assuming Yukawa anarchy, also effect other observables. We also find that the general T -dependence is in agreement with the expected $1/T^2$ behaviour from power-counting.

Next, we turn to the custodially protected model. We now need to include the term with the novel Yukawa Y^u in (31). If both Yukawas were the same, the different contributions would cancel in the dominant term and the total contribution would be small compared to Δa_μ^g . However, there is generally no argument that can be used to fix the relative size of the Yukawa matrices. In the following we assume that the two matrices have a common entry size. The right panel of figure 7 shows the Higgs contribution to

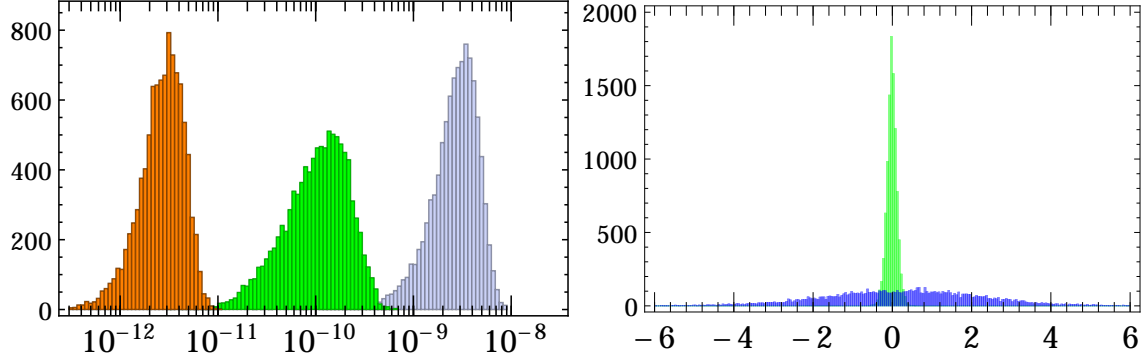


Figure 7: *left panel:* Higgs contribution to $g_\mu - 2$ for different average Yukawa sizes in the minimal model (see text for details). The red (light grey) histogram corresponds to $|Y| \in (1/10, 1/3)$, the green (grey) one to $|Y| \in (1/3, 3)$ and blue (dark grey) to $|Y| \in (3, 10)$. We use $T = 1$ TeV everywhere. *right panel:* As left panel but for the custodially protected model and without the histogram for small Yukawa matrices. The x-axis uses a linear scale in units of 10^{-9} .

the anomalous moment. We only show the two cases of large and intermediate Yukawa entries; on a linear scale the histogram for small Yukawas entries is too narrowly centred around zero to be visible.

As in the minimal scenario we find potentially very large corrections to $g_\mu - 2$ for $T = 1$ TeV. The effect for different choices of T can again be obtained by making use of the overall $1/T^2$ scaling. The preference for a positive contribution that is present in the minimal model is not observed. However, we have to be careful if we want to make statements about possible size of the Higgs contributions. It is well-known that the dipole operator enters not only in the anomalous magnetic moment but also various other observables in lepton flavour physics; notably it gives a potentially large contribution to $\mu \rightarrow e\gamma$. Note, that the gauge contribution to $g_\mu - 2$ is basically independent of the Yukawa structure and the contributions to off-diagonal flavour-changing transitions are suppressed. Hence, experimental bounds on FCNCs will be less important there.

A complete analysis of lepton flavour observables in the RS model will be presented elsewhere. In the following we just illustrate their potential impact by considering the dipole operator contribution to $\mu \rightarrow e\gamma$ alone. Since $\mu \rightarrow e\gamma$ can be mediated by the coefficients α_{12} and α_{21} in the same way as $(g - 2)_\mu$ is mediated by α_{22} , it is straightforward to estimate the branching fraction. A naive estimate that assumes that the dipole operators are essentially structureless in flavour space was already given in [23, 30]. Under this assumption the bounds on FCNCs all but eliminate the Higgs contribution to $g_\mu - 2$. However, especially in the custodial model where we observe two competing contributions due to the presence of an additional Yukawa matrix, this picture may be too simple. In the following we will use the upper bound of 5.7×10^{13} for the $\mu \rightarrow e\gamma$ branching ratio [33]. We again generate random parameter points and require that the dipole operator contribution to $\mu \rightarrow e\gamma$ alone does not violate this bound.

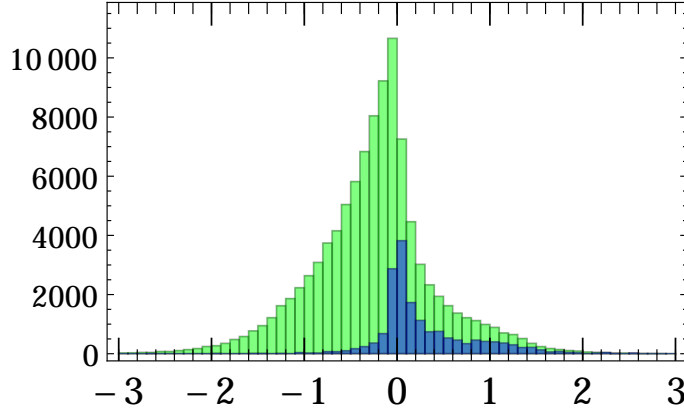


Figure 8: Histogram of the Higgs contribution to $g_\mu - 2$ in units of 10^{-10} for $T = 4$ TeV. Without (green/light grey) and with (blue/dark grey) taking the $\mu \rightarrow e\gamma$ branching ratio into consideration. The height of the blue histogram has been increased by a factor of ten.

In figure 8 we show effect of a^H on $g_\mu - 2$ with (in blue) and without (in green) the $\mu \rightarrow e\gamma$ bound for $T = 4$ TeV. The Yukawa matrix elements were allowed to take values in the interval $(1/10, 10)$. The green (light grey) histogram consists of 10^5 parameter sets, while only ≈ 2000 sets survive the MEG bound. After taking the bound into account, we find a preference for positive contributions a_μ^H as was already the case in the minimal model. The asymmetry arises because the terms with a Y^u factor in (31) tend to generate larger off-diagonal elements in a_{ij}^H than the terms with only one type of Yukawa matrix. Hence, the second term in the third line of (31) is affected more strongly than the predominantly positive first term. For $T = 1$ TeV the $\mu \rightarrow e\gamma$ bound has an even more pronounced effect: if one e.g. restricts the Yukawa entries to the interval $(3, 10)$ no parameter sets that pass the MEG bound were found in a random sample of 10^5 sets and only one in 4000 parameter sets with Yukawas in the interval $(0.1, 10)$ passed the constraint for $T = 1$ TeV. We have to stress, that the random scans through parameter space can only give indications to which extent the contributions to $g_\mu - 2$ are constrained. For a more definite statements a comprehensive analysis of lepton (flavour) observables in the RS model would be necessary.

5 Conclusions

We presented the computation of the leading effects of the custodially protected Randall-Sundrum model on the anomalous magnetic moment of the muon. To this end we extended the techniques developed in [23] for the case of the minimal RS model. We find that the contribution to a_μ mediated by an exchange of a virtual gauge boson is given

by

$$\Delta a_\mu = 2.72 \cdot 10^{-10} \left(\frac{1 \text{ TeV}}{T} \right)^2. \quad (36)$$

This number is essentially independent of the model parameters in the lepton sector and by more that a factor of three larger than the corresponding result in the minimal RS scenario. Despite the enhancement the effect is still insufficient to reconcile theory and experiment.

The effect of diagrams with a virtual Higgs boson can be determined analytically, but requires a precise specification of the localisation of the Higgs near the IR brane. The result (31) is valid for the so-called narrow bulk Higgs (in the language of [16]). Since the Higgs contribution exhibits by definition a strong dependence on the model parameters in the Yukawa sector, one can use e.g. $\mu \rightarrow e\gamma$ to limit the magnitude of the dipole operators, however, such bound can always be circumvented by imposing a specific flavour structure already in the 5D Lagrangian. Without bounds from lepton flavour physics the Higgs contribution can reach values as large as

$$|\Delta a_\mu^H| \lesssim \text{few} \times 10^{-9} \times \left(\frac{1 \text{ TeV}}{T} \right)^2 \quad (37)$$

if the entries of the dimensionless 5D Yukawa matrices are allowed to have magnitudes of up to 10. In the minimal model the contribution is predominantly positive, in the custodially protected model the sign is undetermined. While the inclusion of the current bound on $\mu \rightarrow e\gamma$ seems introduce a preference for a positive shift in $g_\mu - 2$, this has to be studied in more detail and with a larger set of potentially sensitive lepton flavour observables to make a definite statement.

Acknowledgements: We thank M. Beneke for many helpful discussions. The work of P.M. is supported in part by the Gottfried Wilhelm Leibniz programme of the Deutsche Forschungsgemeinschaft (DFG). The work J.R. is supported by STFC UK. The Feynman diagrams were drawn with the help of Axodraw [34] and JaxoDraw [35].

References

- [1] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86** (2012) 010001.
- [2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [hep-ph/9905221].
- [3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690 [hep-th/9906064].
- [4] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Lett. B **473** (2000) 43 [hep-ph/9911262].

- [5] A. Pomarol, Phys. Lett. B **486** (2000) 153 [hep-ph/9911294].
- [6] Y. Grossman and M. Neubert, Phys. Lett. B **474** (2000) 361 [hep-ph/9912408].
- [7] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, Phys. Rev. D **62** (2000) 084025 [hep-ph/9912498].
- [8] S. J. Huber and Q. Shafi, Phys. Lett. B **498** (2001) 256 [hep-ph/0010195].
- [9] K. Agashe, A. Delgado, M. J. May and R. Sundrum, JHEP **0308** (2003) 050 [hep-ph/0308036].
- [10] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, Phys. Lett. B **641** (2006) 62 [hep-ph/0605341].
- [11] CMS Collab., CMS Note PAS-B2G-12-006. ATLAS Collab., ATLAS Note CONF-2013-052.
- [12] M. S. Carena, E. Ponton, J. Santiago and C. E. M. Wagner, Phys. Rev. D **76** (2007) 035006 [hep-ph/0701055].
- [13] M. E. Albrecht, M. Blanke, A. J. Buras, B. Duling and K. Gemmler, JHEP **0909** (2009) 064 [arXiv:0903.2415 [hep-ph]].
- [14] S. Casagrande, F. Goertz, U. Haisch, M. Neubert and T. Pfoh, JHEP **1009** (2010) 014 [arXiv:1005.4315 [hep-ph]].
- [15] J. Hahn, C. Hörner, R. Malm, M. Neubert, K. Novotny and C. Schmell, arXiv:1312.5731 [hep-ph].
- [16] R. Malm, M. Neubert, K. Novotny and C. Schmell, arXiv:1303.5702 [hep-ph].
- [17] M. Carena, S. Casagrande, F. Goertz, U. Haisch and M. Neubert, JHEP **1208** (2012) 156 [arXiv:1204.0008 [hep-ph]].
- [18] A. Azatov, M. Toharia and L. Zhu, Phys. Rev. D **82**, 056004 (2010) [arXiv:1006.5939 [hep-ph]].
- [19] C. Csaki, Y. Grossman, P. Tanedo and Y. Tsai, Phys. Rev. D **83** (2011) 073002 [arXiv:1004.2037 [hep-ph]].
- [20] O. Gedalia, G. Isidori and G. Perez, Phys. Lett. B **682** (2009) 200 [arXiv:0905.3264 [hep-ph]].
- [21] C. Delaunay, J. F. Kamenik, G. Perez and L. Randall, JHEP **1301** (2013) 027 [arXiv:1207.0474 [hep-ph]].
- [22] M. Blanke, B. Shakya, P. Tanedo and Y. Tsai, JHEP **1208** (2012) 038 [arXiv:1203.6650 [hep-ph]].

- [23] M. Beneke, P. Dey and J. Rohrwild, JHEP **1308** (2013) 010 [arXiv:1209.5897 [hep-ph]].
- [24] R. Alonso, E. E. Jenkins, A. V. Manohar and M. Trott, arXiv:1312.2014 [hep-ph].
- [25] F. Ledroit, G. Moreau and J. Morel, JHEP **0709** (2007) 071 [hep-ph/0703262 [HEP-PH]].
- [26] B. Duling, PhD Thesis Technische Universität München (2010)
- [27] W. Buchmüller and D. Wyler, Nucl. Phys. B **268** (1986) 621.
- [28] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010** (2010) 085 [arXiv:1008.4884 [hep-ph]].
- [29] L. Randall, M. D. Schwartz, JHEP **0111** (2001) 003, hep-th/0108114.
- [30] M. Beneke, P. Moch and J. Rohrwild, arXiv:1404.7157 [hep-ph].
- [31] T. Hahn, Comput. Phys. Commun. **168** (2005) 78 [hep-ph/0404043].
- [32] A. Crivellin, S. Najjari and J. Rosiek, arXiv:1312.0634 [hep-ph].
- [33] J. Adam *et al.* [MEG Collaboration], Phys. Rev. Lett. **110** (2013) 201801 [arXiv:1303.0754 [hep-ex]].
- [34] J. A. M. Vermaseren, Comput. Phys. Commun. **83** (1994) 45.
- [35] D. Binosi and L. Theussl, Comput. Phys. Commun. **161** (2004) 76 [hep-ph/0309015].